Chapter 5 : Playing with Numbers

ANSWER KEYS

EXERCISE 5.1

1. Generalised form:

(*i*) 85 = 10 × 8 + 5

(*ii*) 398 = 100 × 3 + 10 × 9 + 8

(*iii*) 726 = 100 × 7 + 10 × 2 + 6

(*iv*) 545 = 100 × 5 × 10 × 4 + 5

2. Usual form:

(*i*) 10 × 6 + 7 = 60 + 7 = 67

(*ii*) 100 × 5 + 4 × 10 + 9 = 500 + 40 + 9 = 549

(*iii*) $100 \times x + 10xy + z = 100x + 10y + z = xyz$

3. 95 and 59 are the numbers formed by the digits 5 and 9.

(*i*) We know that, $ab - ba$ when divided by 9, the quotient is $(a - b)$. (rule 2)

∴ 95 – 59 when divided by 9, the quotient is 9 – 5 = 4.

(*ii*) $ab - ba$ when divided by $(a - b)$ the quotient is 9.

∴ 95 – 59 when divided by (9 – 5) *i.e.*, 4, the quotient is 9.

4. 79 and 97 are the numbers formed by the digits 7 and 9.

(*i*) If the sum of these numbers (79 + 97) is divided by 11, we get $\frac{79+97}{11} = \frac{176}{11}$ = 16 as the quotient.

(*ii*) If the sum of the numbers (79 + 97) is divided by the sum of digits (7 + 9) = 16, we get

$$\frac{97+97}{7+9} = \frac{176}{16} = 11 \text{ as quotient.}$$

5. The given number is 784.

Other two numbers obtained by arranging the digits of 784 in cyclic order are 847 and 478.

The sum of the numbers = 84 + 847 + 478
= 2109

The sum of the digits 7, 8 and 4 = 7 + 8 + 4 = 19.

(*i*) When divided by 111, gives the quotient 19.

$$\therefore \quad \frac{2109}{111} = 19$$

(*ii*) When divided by 37, gives the quotient 3 × sum of the digits

= 3 × 19 = 57

$$\therefore \quad \frac{2109}{37} = 57$$

(*iii*) When divided by 19 (sum of the digits), gives the quotient 111.

$$\therefore \quad \frac{2109}{19} = 111.$$

6. Let $abc = 952$ and $cba = 259$

$$\begin{aligned} abc - cba &= 99\,(a - c) \\ &= 99\,(a - c) \\ &= 9 \times 11\,(a - c) \\ &= 9 \times 11\,(a - c) \end{aligned}$$

$$\begin{aligned} \therefore \quad \frac{abc - cba}{9} &= \frac{9 \times 11\,(a-c)}{9} \\ &= 11\,(a - c) \\ &= 11 \times (9 - 2) \\ &= 11 \times 7 = 77. \end{aligned}$$

Hence, the required quotient is 77.

EXERCISE 5.2

1. ∵ Sum of the numbers in each row, each column, and each diagonal is 15. Therefore,

C_1	C_2	C_3	
6	**1**	8	← R_1
7	5	**3**	← R_2
2	**9**	4	← R_3

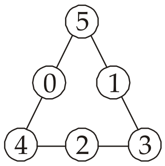
2.

C_1	C_2	C_3	
13	14	9	← R_1
8	12	16	← R_2
15	10	11	← R_3

3.

C_1	C_2	C_3	
13	**6**	**11**	← R_1
8	**10**	**12**	← R_2
9	**14**	**7**	← R_3

4. The given numbers are 0, 1, 2, 3, 4, 5 and 12. Since, each side of triangle add up to 9.



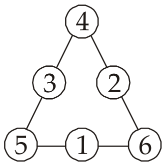
Therefore,

(i) 5 + 0 + 4 = 9

(ii) 4 + 2 + 3 = 9

(iii) 5 + 1 + 3 = 9

5. The given numbers are 1, 2, 3, 4, 5 and 6. Since, each side of triangle add upto 12.



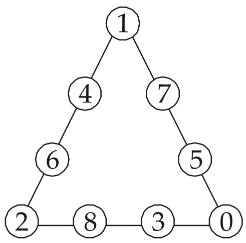
Therefore,

(i) 4 + 3 + 5 = 12

(ii) 5 + 1 + 6 = 12

(iii) 6 + 2 + 4 = 12

6. The given numbers are 0, 1, 2, 3, 4, 5, 6, 7 and 8. Since each side of a triangle adds up to 13.



Therefore,

(i) 1 + 4 + 6 + 2 = 13

(ii) 2 + 8 + 3 + 0 = 13

(iii) 0 + 5 + 7 + 1 = 13

EXERCISE 5.3

1. ∵ A number is divisible by 3, if the sum of digits of that number is divisible by 3. Therefore,

(i) The sum of digits of number 616 is 6 + 1 + 6 = 13, which is not divisible by 3.

So, the number 616 is not divisible by 3.

(ii) The sum of digits of number 537 is (5 + 3 + 7) = 15, which is divisible by 3.

So, the number 537 is divisible by 3.

(iii) The sum of digits of number 927 is (9 + 2 + 7) = 18, which is divisible by 3.

So, the number 927 is divisible by 3.

Similarly (iii) 924, and (vi) 543 also divisible by 3.

Hence, (ii), (iii), (iv) and (vi) divisible by 3.

2. ∵ If the sum of digits of the given number is divisible by 9, then the given number is divisible by 9.

(i) The sum of digits of number 199 is (1 + 9 + 9) = 19, which is not divisible by 9.

So, the number 199 is not divisible by 9.

(ii) The sum of digits of number 477 is (4 + 7 + 7) = 18, which is divisible by 9.

So, the number 477 is divisible by 9.

Similarly, (iii) 2349 and (iv) 1989 also divisible by 9.

Hence, (ii), (iii) and (iv) are divisible by 9.

3. For a number to be divisible 11, (sum of digits at odd places) – (sum of digits at even places) = 0 or a multiple of 11. Therefore,

(i) 264

Sum of digits at odd places = 4 + 2 = 6

Sum of digits at even places = 6

Difference between two sums = 6 – 6 = 0

So, the number 264 is divisible by 11.

(ii) 968

Sum of digits at odd places = 8 + 9 = 17

Sum of digits at even places = 6

Difference between two sums = 17 – 6 = 11

Which is multiple of 11.

So, the number 968 is divisible by 11.

(iii) 61809

Sum of digits at odd places = 9 + 8 + 6 = 23

Sum of digits at even places = 0 + 1 = 1

Difference between two sums = 23 – 1 = 22

Which is a multiple of 11.

So, the number 61809 is divisible by 11.

Similarly (iv) 32571 also divisible by 11.

Hence, (i), (ii), (iii) and (iv) are divisible by 11 and (v), (vi) are not divisible by 11.

4. ∵ 31x is a multiple of 11.

∴ $(x + 3) - 1 = 0$ or multiple of 11.

∴ $x + 2$ can takes values from 0, 11, 22, 33, ... Since x is a digit, for the value of digit x.

$$x + 2 = 11$$

$\Rightarrow$ $x = 11 - 2 = 9$ $\Rightarrow$ $x = 9$.

5. $\because$ $21y5$ is a multiple of 9.

$\therefore$ $2 + 1 + y + 5 = 8 + y$ is a multiple of 9.

$\therefore$ $8 + y$ can take values 0, 9, 18, 27, ...

Since, y is a digit. Therefore,

$$8 + y = 9 \Rightarrow y = 9 - 8$$

$$\Rightarrow y = 1.$$

6. $\because$ A number is divisible by 3, if the sum of digit of number is divisible by 3.

$\therefore$ The sum of the digits of number 32699 is 3 + 2 + 6 + 9 + 9 = 29 = (27 + 2).

If 2 is subtracted from the number the rest number is exactly divisible by 3.

Hence, remainder = 2.

7. $\because$ $68y5$ is divisible by 11.

$\therefore$ $(5 + 8) - (y + 6)$ is 0 or multiples of 11.

$\therefore$ $7 - y$ can take values 0, 11, 22, ...

Since y is a digit. Therefore,

$$7 - y = 0 \Rightarrow \boxed{y = 7}$$

8. $\because$ $13y8$ is divisible by 6 *i.e.*, it is divisible by 2 and 3.

$\therefore$ $1 + 3 + y + 8$ is divisible by 3, 6, 9, 12, ...

$\therefore$ $12 + y$ can take value 3, 6, 9, 12, 15 .... Since y is a digit.

$\Rightarrow$ $12 + y = 12, 15, 18, 21$

$\Rightarrow$ $y = 0, 3, 6, 9.$

9. The sum of digits of number 524387 is (5 + 2 + 4 + 3 + 8 + 7) = 29 which is not divisible by 3.

$\therefore$ 29 = 27 + 2, if 2 is subtract from the numbers the rest number is exactly divisible by 3.

Hence, required remainder = 2.

EXERCISE 5.4

1.
$$\begin{array}{r} 3\ 1\ Q \\ +\ 1\ Q\ 3 \\ \hline 5\ 0\ 1 \end{array}$$

If Q replaced with 8, then 8 + 3 = 11, gives 1 as units digit in the sum and 1 + 1 + 8 = 10 gives 0 as tens place in the given sum.

Hence, $\boxed{Q = 8}$.

2.
$$\begin{array}{r} 2\ A\ B \\ A\ B\ 1 \\ \hline B\ 1\ 8 \end{array}$$

$\Rightarrow$ $B + 1 = 8 \Rightarrow B = 7$

and 4 + 7 = 11, gives 1 as tens place in the sum.

Hence, $\boxed{A = 4, B = 7}$.

3.
$$\begin{array}{r} 8\ A\ 5 \\ +\ 9\ 4\ A \\ \hline 1A\ 3\ 3 \end{array}$$

For, 3 as units digit in the given sum, $5 + A$ should be 13.

$\Rightarrow$ $5 + A = 13$

$\Rightarrow$ $\boxed{A = 8}$

4.
$$\begin{array}{r} A\ B \\ \times\ 5 \\ \hline C\ A\ B \end{array}$$

If we take $B = 5$. Then 5 × 5 = 25, gives 5 at units digit as B.

And if we take $A = 2$, then 2 × 5 = 10

$\therefore$ 10 + 2 = 12, gives 2 as A at tens place of the product. Therefore,

$A = 2$, $B = 5$, $C = 1$.

MULTIPLE CHOICE QUESTION

1. Sum of number 98 + 89 = 187

Sum of their digits = 9 + 8 = 17

Therefore, required quotient = $\frac{187}{17} = 11$

Hence, option (*a*) is correct.

$\because$ If a number is divisible by 3, the sum of digit of the number is divisible by 3. Therefore,

Sum of digits of number 543 is 5 + 4 + 3 = 12 is divisible by 3.

Sum of digit of number 499 is 4 + 9 + 9 = 22 is not divisible by 3.

Sum of digit of number 785 is 7 + 8 + 5 = 20 is not divisible by 3.

Sum of digit 648 is 6 + 4 + 8 = 18 is divisible by 3.

Thus, (*b*) 499 and (*c*) 785 is not divisible by 3.

Hence, option (*b*) and (*c*) is correct.

3. $\because$ $(ab + ba)$ is always a multiple of 11.

Hence, option (*d*) is correct.

4. If a number is divisible by 9, if the sum of digits is divisible by 9.

$\therefore$ 927 is divisible of 9, since sum of digits

9 + 2 + 7 = 18 is divisible by 9.

Hence, option (*b*) is correct.

5. $\because$ $\overline{31y6}$ is divisible by 3.

$\therefore$ $3 + 1 + y + 6$ is a multiple of 3.

$\therefore$ $10 + y$ can take values from 0, 3, 6, 9, 12, ...

Since, y is a digit, for least value of y.

$$10 + y = 12 \Rightarrow y = 2$$

Hence, option (*c*) is correct.

6. $\because$ If the units digit of a number is 0, then the number is divisible by 5 and 10.

$\therefore$ 2640 is a number which is divisible by 5 and 10.

Hence, option (*b*) is correct.

7. *abc* – *cba* is always a multiple of 9 and 11.

Hence, option (*a*) and (*b*) is correct.

8. $\because$ $\overline{24x}$ is divisible by 9.

$\therefore$ $(2 + 4 + x)$ is a multiple of 9.

$\therefore$ $6 + x$ can take values from 9, 18, 27, ...

Since, x is a digit, for value of x.

$$6 + x = 9 \Rightarrow x = 3$$

Hence, option (*c*) is correct.

9. $A + A + A = BA$

If $5 + 5 + 5 = 15$

Thus, $A = 5$, $B = 1$

Hence, option (*b*) is correct.

MENTAL MATHS CORNER

A. Fill in the blanks:

1. The quotient when the difference of 785 – 758 is divided by 9 is **3**.
2. If we subtract the ones digit of a number from that number, the remainder is always divisible by **2, 5** and **10**.
3. The three whole numbers whose product and sum are equal are **1, 2** and **3**.

   $\because$ $1 + 2 + 3 = 6$ and $1 \times 2 \times 3 = 6$
4. The two numbers whose product is a one-digit number and the sum is a two-digit number are **1** and **9**.

B. 'True' or 'False'

1. If a number is divisible by 3, it must be divisible by 9. **(False)**
2. If a number divides three numbers exactly, it must divided their sum exactly. **(True)**
3. A number is divisible by 18, if it is divisible by both 3 and 6. **(False)**
4. The sum of two consecutive odd numbers is always divisible by 4. **(True)**
5. If a number is divisible by any number *p*, then it will also be divisible by each of the factor of *p*. **(True)**

REVIEW EXERCISE

1. (*i*) $56 = 5 \times 10 + 6$

(*ii*) $79 = 7 \times 10 + 9$

(*iii*) $43 = 4 \times 10 + 3$

(*iv*) $92 = 9 \times 10 + 2$

2. (*i*) $549 = 5 \times 100 + 4 \times 10 + 9$

(*ii*) $768 = 7 \times 100 + 6 \times 10 + 8$

(*iii*) $832 = 8 \times 100 + 3 \times 10 + 2$

(*iv*) $996 = 9 \times 100 + 9 \times 10 + 6$

3. $\because$ $31z5$ is a multiple of 3.

$\therefore$ $(3 + 1 + z + 5) = 9 + 2$ is a multiple of 3.

$\therefore$ $9 + z$ can take values from 0, 3, 6, 9, 12, 15, ...

Since, z is a digit. Therefore,

$$9 + z = 9, 12, 15, 18$$

$\Rightarrow$ $z = 0, 3, 6, 9.$

4. (*i*)
$$\begin{array}{r} 3\ A \\ +\ 2\ 5 \\ \hline B\ 2 \end{array}$$

If $7 + 5 = 12$, gives 2 as units digit.

Therefore, $\boxed{A = 7}$

Now, $37 + 25 = 62$, therefore, $B = 6$.

Hence, $\boxed{A = 7, B = 6}$

(*ii*)
$$\begin{array}{r} 9\ A \\ \times\ A \\ \hline 9\ A \end{array}$$

Here, the ones digit of $A \times A = A$, it must be that $A = 1$, 5 or 6.

$A = 1, 5$ satisfy $A \times A = A$, but it is not possible as the product is $9A$.

Now, $A = 6$, we have

$$\begin{array}{r} 1\ 6 \\ 6 \\ \hline 9\ 6 \end{array}$$

Clearly, $A = 6$, satisfy the give product.

Hence, $\boxed{A = 6}$

5. Let *ab* be the two digit number and *ba* is the number obtaining reversing the digit. Therefore,

$$ab - ba = 9 \times (a - b)$$

$\because$ $ab - ba = 18$

Therefore,

$$9 \times (a - b) = 18$$

$$a - b = \frac{18}{9} = 2$$

$\therefore$ $a - b = 2$

Hence, the difference between the digits of the number is 2.

6.

	C_1	C_2	C_3	
	8	**1**	**6**	$\leftarrow R_1$
	3	5	7	$\leftarrow R_2$
	4	**9**	2	$\leftarrow R_3$

$\because$ $\overline{49y\,17}$ is divisible by 9.

$\therefore$ $4 + 9 + y + 1 + 7 = 21 + y$ is a multiple of 9.

$21 + y$ can take values from 9, 18, 27, ...

Since y is a digit. Therefore

$$21 + y = 27 \quad \Rightarrow \quad y = 6$$

Hence, $\boxed{y = 6}$

HOTS QUESTIONS

1. *abc* and *cab* be the three digit numbers.

$$\begin{aligned} abc &= a \times 100 + b \times 10 + c \\ &= 100a + 10b + c \\ bca &= b \times 100 + c \times 10 + a \\ &= 100b + 10c + a \\ cab &= c \times 100 + a \times 10 + b \\ &= 100c + 10a + b \end{aligned}$$

Now,

$$\begin{aligned} abc + bca + cab &= 111a + 111b + 111c \\ &= 111(a + b + c) \\ abc + bca + cab &= 111 \times (a + b + c) \end{aligned}$$

Hence, the sum of $abc + bca + cab$ is divisible by 111.

2.

$$\begin{array}{r} A\;\;B\;\;C\;\;D \\ \times\;4 \\ \hline D\;\;C\;\;B\;\;A \end{array}$$

If we put $D = 8$. Therefore, $8 \times 4 = 32$, gives 2 as units place. And $2 \times 4 = 8$.

Thus, $\boxed{A = 2}$

If $C = 7$. Therefore, $7 \times 4 = 28$ and $28 + 3 = 31$ gives 1 as tens place.

If $B = 1$. Therefore, $1 \times 4 = 4$ and $4 + 3 = 7$ as hundreds place.

Hence, $A = 2$, $B = 1$, $C = 7$, $D = 8$.

Puzzle

Let the 2-digit number be *ab*.

$\therefore$ $ab = 10a + b$ ...(*i*)

If we multiply *ab* by 9 we get a 3-digit number say *dce*.

$dce = 9(ab)$

$$100d + 10c + e = 9(10a + b) \qquad ...(ii)$$

If 4 is written before the 3-digit number and 6 is written after the 3-digit number we get a 5-digit number. *i.e.*, 4*dce*6.

As per given condition,

$$4dce6 = 42355 + dce$$

$$40000 + 1000d + 100c + 10e + 6 = 42355 + 100d + 10c + e$$

$$40006 + 10(100d + 10c + e) = 42355 + (100d + 10c + e)$$

$$40006 + 10 \times 9(10a + 6) = 42355 + 9(10a + 6)$$

from (*ii*)

$$\begin{aligned} 90(10a + 6) - 9(10a + 6) &= 2349 \\ 81(10a + b) &= 2349 \\ 10a + b &= 29 \end{aligned}$$

or $ab = 29$ from (*i*)

My age is 29 years.